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ON THE NATURE OF THE ANOMALOUSLY SLOW APSIDAL MOTION OF DI HERCULIS

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ABSTRACT

The elliptical three-body problem (a close binary and a nearby companion, three-dimensional case) is examined to elucidate the nature of anomalies in apsidal motion of some close binary systems. The numerical results are presented for once- and twice-averaged problems. It is shown that the significant discrepancy between the observed and the theoretical apsidal motion of DI Her (relativistic and classical effects) possibly results from the close binary orbit perturbations due to a third body.

Subject headings: stars: eclipsing binaries — stars: individual (DI Herculis)

1. INTRODUCTION

The apsidal motion rate (AMR) in a close binary system (CBS) depends upon the density distribution inside the components, and that can be easily computed for theoretical stellar models. The AMR observations in eclipsing binaries provide an excellent opportunity to check the general results against the stellar structure theory. The relation of the apsidal motion constant k_2 to the initial chemical composition, masses, and ages of the evolving main-sequence stars has been investigated by many authors (Semeniuk & Paczyński 1968; Cisneros-Parra 1970; Petty 1973; Odell 1974; Stothers 1974; Monet 1980; Giménez & Garcia-Pelayo 1982; Jeffery 1984; Hejlesen 1987) and is a good example of recent progress in this field. The observational rates of the apsidal motion (and the corresponding k_2 value) determined for several dozens of CBSs are in reasonable agreement with the theoretical values. Obviously this agreement will be improved with the improvement of both stellar structure theory and observational methods. There are, however, well-observed eclipsing binaries exhibiting significant discrepancies between the observed and the predicted AMR. One of them is DI Her (HD 175227; Sp: B4 V + B5 V, $P = 10.55$ days). The DI Her system is interesting because of the large orbital eccentricity ($e = 0.48$) and the significant displacement of the secondary minimum (ϕII = 0.77). Moreover, the relativistic term $\mathrm{AMR}^{\mathrm{th}}_{\mathrm{rel}}$ should predominate in the periastron motion of DI Her because of the large mass ($\mathfrak{M}_1 + \mathfrak{M}_2 \simeq 10\ \mathfrak{M}_\odot$) and small fractional radii ($r \simeq 0.06$) of the components (Rudkjøbing 1959). Hence this system is a favorable object for testing the predictions of general relativity (GR).

Unfortunately, because of the low accuracy of early visual and photographic observations, reliable measurements of the apsidal motion rate of DI Her were not available for a long time. Martynov & Khaliullin (1978, 1980), using their own and Semeniuk's (1968) multicolor photoelectric data, obtained an unexpected result: the observed apsidal motion rate of DI Her ($\mathrm{AMR}^{\mathrm{obs}} = 0\overset{\circ}{.}0124\ \mathrm{yr}^{-1}$) is less than one-third of the theoretical value ($\mathrm{AMR}^{\mathrm{th}} = 0\overset{\circ}{.}0426\ \mathrm{yr}^{-1}$) predicted by the combined general relativistic ($\mathrm{AMR}^{\mathrm{th}}_{\mathrm{rel}} = 0\overset{\circ}{.}0233\ \mathrm{yr}^{-1}$) and classical (tidal-rotational) ($\mathrm{AMR}^{\mathrm{th}}_{\mathrm{cl}} = 0\overset{\circ}{.}0193\ \mathrm{yr}^{-1}$) effects. Because the accuracy of the simultaneous determination of all orbital elements from a light-curve solution is not high enough to give the rate of the periastron advance directly, the only appropriate way to evaluate $\mathrm{AMR}^{\mathrm{obs}}$ is to assume that all orbital elements except the longitude of periastron ω are time-independent and to investigate the phase variation of the secondary minimum or the difference of the periods: $\Delta P^{\mathrm{obs}} = P_2 - P_1$ (Rudkjøbing 1959). The observed value of $\Delta P^{\mathrm{obs}} = 0\overset{\mathrm{s}}{.}61 \pm 0\overset{\mathrm{s}}{.}09$ obtained by Martynov & Khaliullin (1980) (corresponding to $\mathrm{AMR}^{\mathrm{obs}} = 0\overset{\circ}{.}0124\ \mathrm{yr}^{-1}$) was confirmed recently by high-accuracy photoelectric observations (Diethelm 1986; Khodykin & Volkov 1989). Reisenberger & Guinan (1989) find $\mathrm{AMR}^{\mathrm{obs}} = 0\overset{\circ}{.}010 \pm 0\overset{\circ}{.}003\ \mathrm{yr}^{-1}$ using all the available data up to 1985.

To solve the puzzle of DI Her, several hypotheses have been considered (see also Guinan & Maloney 1985). Moffat (1984, 1989) proposed to investigate the apsidal motion in DI Her in terms of the nonsymmetric gravitational theory—an alternative to GR. Shakura (1985) and Company, Portillo, & Giménez (1988) have suggested that a low rate of the periastron motion may be explained by rapid axial rotation of one or both components, whose rotational axes are highly inclined from the normal to the orbital plane. In this case the precession of the axes of the stars can occur. Reisenberger & Guinan (1989) present very weak evidence supporting this effect, but we think the differences in the measured values of $V \sin i_{\mathrm{rot}}$ with time are probably due to observational errors. As for resonance effects involved in tidal interaction of the components (Papaloizou & Pringle 1980), this phenomenon seems unlikely to be significant for such a long-period system as DI Her. Hegedüs & Nuspl (1986) tried to explain the anomalies of the AMR in DI Her by orbital precession. However, Khodykin (1989) showed that the orbital plane precession of an eclipsing binary is unable to distort significantly the $\mathrm{AMR}^{\mathrm{obs}}$. Martynov and Khaliullin and Guinan and Maloney investigated the third-body hypothesis. Only the variations of the periastron longitude have been considered using the Brown's relationships (Guinan & Maloney 1985), and the parameters of such a triple system providing the desired periastron regression were found. However, this solution does not satisfy either the triple system stability criteria or the boundary conditions for light-curve distortion.

At the same time, Martynov & Khaliullin (1980) attempted to explain the observed variation of ϕII in the frame of the two-body model by assuming that both the periastron longitude and the orbital eccentricity are not constant in time. It has been shown that the eccentricity decrease $de/dt = -8 \times 10^{-5}\ \mathrm{yr}^{-1}$ would be enough to eliminate the discrepancy, but such rapid circularization contradicts the binary age ($t \simeq 5 \times 10^7$ yr) and, on the other hand, cannot result from classical tidal effects (Zahn 1977). The value of t_{circ} for DI Her exceeds the hydrogen core depletion time.

The purpose of our investigations is to explain the anomalies in apsidal motion of DI Her by taking into account the third-body influence on *all* the orbital elements. Such an influence should result in the systematic relative shift of the secondary minimum giving the difference of the periods: $\Delta P^{\mathrm{tb}} = \Delta P^{\mathrm{obs}} - \Delta P^{\mathrm{th}}$. The theoretically expected value of $\Delta P^{\mathrm{th}} = 2\overset{\mathrm{s}}{.}29 \pm 0\overset{\mathrm{s}}{.}12$, corresponding to $\mathrm{AMR}^{\mathrm{th}} = 0\overset{\circ}{.}0426 \pm 0\overset{\circ}{.}0030$ yr^{-1}, can be found from Rudkjøbing's relation. Since $\Delta P^{\mathrm{obs}} = 0\overset{\mathrm{s}}{.}61 \pm 0\overset{\mathrm{s}}{.}09$ (see above), the value of ΔP^{tb} is $-1\overset{\mathrm{s}}{.}68 \pm 0\overset{\mathrm{s}}{.}15$. This effect should last during the time t_{eff}, which is at least of the order of the interval of observations $t_{\mathrm{obs}} \simeq 100$ yr (from this point of view Guinan and Maloney's investigation is not complete). By taking into account both photovisual estimations and photoelectric data, we must require that the eccentricity and the orbital inclination vary within the observational errors during t_{obs}. Since an examination of the depths of the eclipses in DI Her reveals no variations greater than 0.05 mag, the change in the inclination is assumed to be less than $0\overset{\circ}{.}5$ over the interval t_{obs}. We assume also that the triple system is stable, and the CBS retains its eccentric orbit for the time of its existence. If the triple system was formed not by exchange or by capture, its age would be equal that of the binary.

2. CHOICE OF THE COORDINATE SYSTEM

Let the origin of the Cartesian coordinate system XYZ coincide with $\mathfrak{M}_1$, with the Z-axis directed toward the observer (see Fig. 1). The elements i_{ph}, Ω_{ph}, ω_{ph} determine the orientation of an eclipsing binary orbit in space: i_{ph} is the orbital inclination relative to the visual plane XY; Ω_{ph} is the longitude of the ascending node, the angle between an arbitrary axis X and the line of nodes $\xi\xi'$ (the line of intersection of the orbital plane and XY); and ω_{ph} is the longitude of periastron π, measured in the orbital plane from ξ. The index "ph" is used because the values of i_{ph}, ω_{ph} can be determined from the analysis of photometric data.

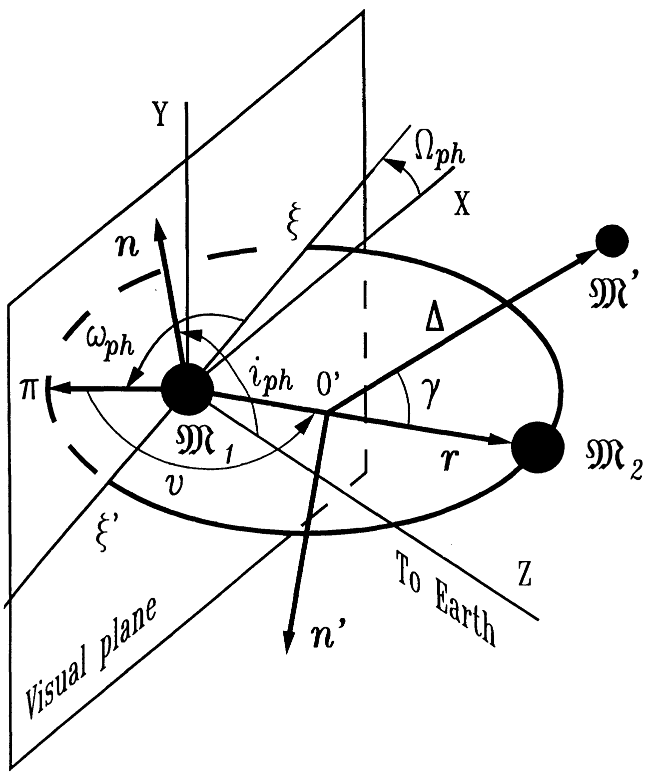

FIG. 1.—Coordinate system and orbital elements used in the paper. The Z-axis is fixed to the line of sight; X is chosen arbitrarily; $\xi\xi'$ is the nodal line (the line of intersection of the orbital plane and the visual plane XY); O' is the barycenter of the eclipsing pair; and n and n' are the normals to the orbital planes.

The theoretical value of $(d\omega_{\mathrm{ph}}/dt)^{\mathrm{th}} = \mathrm{AMR}^{\mathrm{th}}$ (the general relativistic and classical effects) can be computed using well-known relationships (Levi-Civita 1937; Sterne 1939; Barker & O'Connell 1978).

Other coordinate systems and orbital elements can also be used. For example, the main coordinate plane of the so-called dynamical coordinate system can be fixed to the total angular momentum vector $\mathfrak{L}$ of the CBS (the Laplace plane). The orbital elements are i, Ω, ω: the inclination i of the orbital plane toward the Laplace plane (usually very small), the longitude of ascending node Ω measured from the visual plane to the line of nodes which is the line of intersection of the orbital and the Laplace planes, and the periastron longitude ω—the angle between the node line and the line of apsides. Since it is impossible to obtain the values i, Ω, ω from the photometric observations, the dynamical coordinate system is not used in this study. It should be stressed that, in general, the value of $d\bar{\omega}/dt = d(\Omega + \omega)/dt$ but not that of $d\omega/dt$ serves as a good approximation to the AMR. Of course, there is no difference between $d\bar{\omega}/dt$ and $d\omega/dt$ when the nodes are static ($d\Omega/dt = 0$). The theoretical expression for $(d\bar{\omega}/dt)^{\mathrm{th}}$ was obtained by Kopal (1978). However, often the confusion between ω_{ph}, $\bar{\omega}$, and ω leads to incorrect conclusions (Batten 1973). For example, Hegedüs and Nuspl (1986), considering the dynamical coordinate frame, suggested that the nodal regression (the orbital plane precession) distorts the $\mathrm{AMR}^{\mathrm{obs}}$ (i.e., the value of $d\omega_{\mathrm{ph}}/dt$) and may remove the observed discrepancy for DI Her. In reality the precession correction for $d\bar{\omega}/dt$ is of the order of

$$(1 - \cos i)\left|\frac{d\Omega}{dt}\right| \simeq \frac{i^2}{2}\left|\frac{d\Omega}{dt}\right|$$

(since $i < 0\overset{\circ}{.}1$ in the case of DI Her, it must be less than $2 \times 10^{-6}\,|d\Omega/dt|$). As to the precession correction for $d\omega_{\mathrm{ph}}/dt$, it seems to be negligible also:

$$\left|\Delta\left(\frac{d\omega_{\mathrm{ph}}}{dt}\right)_{\mathrm{precs}}\right| \leq \sin i \cos i_{\mathrm{ph}}\left|\frac{d\Omega}{dt}\right|$$

(for DI Her $\sin i \cos i_{\mathrm{ph}} \simeq 2 \times 10^{-5}$, $i_{\mathrm{ph}} = 89\overset{\circ}{.}3$). Therefore, the orbital precession hypothesis is unable to explain the AMR anomalies in eclipsing binaries (Khodykin 1989).

3. THIRD-BODY MODEL

We shall use the parameters of the eclipsing binary from Martynov & Khaliullin (1980) and Popper (1982): $\mathfrak{M}_1 = 5.15$, $\mathfrak{M}_2 = 4.52$ (in solar masses); $P = 10.55017$ days; $e = 0.482$; $i_{\mathrm{ph}} = 89\overset{\circ}{.}3$; $\omega_{\mathrm{ph}} = 151°$ (the periastron longitude of the relative orbit of the secondary component differs from that of the primary one by 180°). L', $\mathfrak{M}'$, P', q', n', e', i'_{ph} correspond to the third component. The upper limit $\mathfrak{M}'$ was found from the mass-luminosity relation to be 1.7 $\mathfrak{M}_{\odot}$ assuming that the luminosity L' is less than $0.03(L_1 + L_2) = 8.1\ L_{\odot}$. It corresponds to the mean mass of A7–A8 main-sequence stars. From the analysis of $O - C$ variations of the photoelectric data (Khodykin & Volkov 1989), the amplitude of the Röemer delay was found to be less than 0.002 days (3 minutes).

4. COMPUTATIONS

First an elliptic three-body problem (S0) at the various mutual orientations of the orbits has been considered. To simplify the model under investigation, the masses have been assumed to be pointlike; the tidal and relativistic effects were ignored, and the third-body motion around the center of gravity of the binary was supposed to be unaffected. The orbital motion of the CBS has been treated as a disturbed Keplerian one. The coordinate system "ph" was used.

Let us consider a binary system with masses of the components $\mathfrak{M}_1$ and $\mathfrak{M}_2$ ($\mathfrak{M}_2 = q\mathfrak{M}_1$, $q < 1$), separated by the radius vector r (see Fig. 1); Δ is the distance between O', the center of gravity of the CBS, and the third body ($\mathfrak{M}' = q'\mathfrak{M}_1$); γ is the angle between r and Δ. The perturbative function for S0 is

$$R = \frac{4\pi^2 q'\mathfrak{M}_1}{\Delta} \sum_{n=2}^{\infty} \frac{1-(-q)^{n-1}}{(1+q)^{n-1}} \left(\frac{r}{\Delta}\right)^n P_n(\cos\gamma) , \tag{1}$$

where $P_n(\cos\gamma)$ are Legendre polynomials of order n. The units of length, time, and mass are AU, years, and $\mathfrak{M}_\odot$, respectively. The expansion of R and its partial derivatives has been evaluated up to $n = 9$, and the variation of orbital elements of the CBS a, e, i_{ph}, Ω_{ph}, ω_{ph}, M for $t \geq P'$ was found. A fourth- to fifth-order Runge-Kutta-Fehlberg scheme (Forsythe, Malcolm, & Moler 1977) was used for the numerical integration of Lagrange equations. The times of minima $T_{1,k}$ and $T_{2,k}$ for the revolution k of the binary were obtained from the condition $\delta(T_{i,k})$ = minimum, where δ is the projection of r on the visual plane, which is obviously true for spherical stars. The effect of the third-body perturbations (the period difference) has been determined from the relation

$$\Delta P^{tb} = (T_{2,k} - T_{2,k-1}) - (T_{1,k} - T_{1,k-1}) . \tag{2}$$

The consideration of the once-averaged elliptic three-body problem (S1) allowed a significant reduction in the computing time. The perturbative function R^* was obtained by averaging R over the orbital period P, since $P \ll P'$:

$$R^* = (2\pi)^{-1} \int_0^{2\pi} R\, dM = \sum_{n=2}^{\infty} R_n^* . \tag{3}$$

Only the terms of order $n = 2, 3$ were used for calculations:

$$R_2^* = \pi^2 q'\mathfrak{M}_1 \left(\frac{a^2}{\Delta^3}\right) \times [3(1+4e^2)I^2 + 3(1-e^2)J^2 - (2+3e^2)] , \tag{4}$$

$$R_3^* = -\frac{5}{4} \pi^2 q'\mathfrak{M}_1 eI \left(\frac{1-q}{1+q}\right)\left(\frac{a^3}{\Delta^4}\right) \times [5(3+4e^2)I^2 + 15(1-e^2)J^2 - 3(4+3e^2)] , \tag{5}$$

where I, J are the cosines of the angles between the radius vector Δ and $r(v)$ at the true anomaly $v = 0°$ and $90°$.

Lagrange equations for osculating elements e, i_{ph}, Ω_{ph}, ω_{ph}, and M (a = constant) have been solved numerically by the above-mentioned scheme. The motion of the CBS during the revolution k was assumed to be unperturbed. The values of the true anomaly v_{ik} corresponding to the times of minima were determined from the condition $\delta(v_{ik})$ = minimum; then mean anomalies M_{ik} and phases of the secondary minimum $\phi \mathrm{II}_k$ were found by successive iterations. Finally, the relation $\Delta P^{tb} = (\phi\mathrm{II}_k - \phi\mathrm{II}_{k-1})P$ gave the difference of the periods.

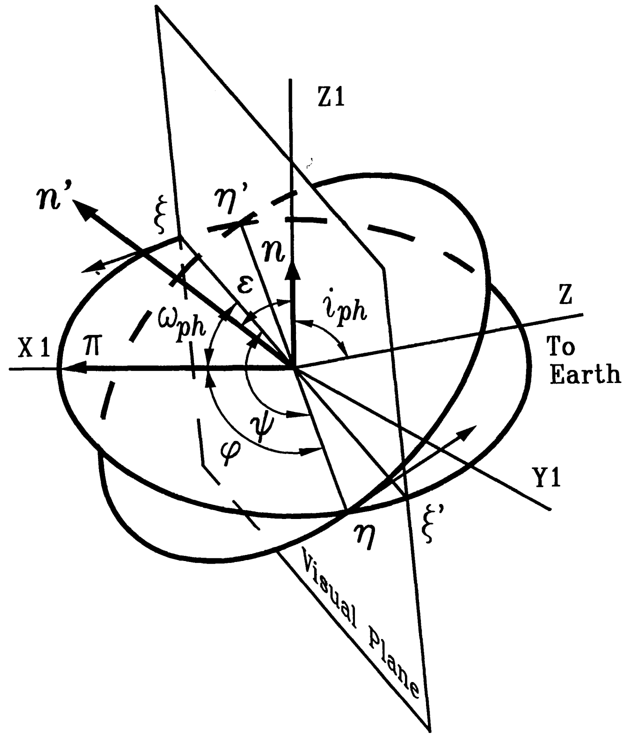

FIG. 2.—The coordinate system $X_1Y_1Z_1$ fixed to the orbital plane of the binary. The projections of the binary's and the third body's orbits on the celestial sphere are shown. The X_1- and Z_1-axes coincide with the radius vector of the periastron π and with the normal of the orbital plane n, respectively; $\xi\xi'$ is the nodal line, and $\eta\eta'$ is the line of intersection of the orbital planes.

To investigate the triple system long-term behavior, the twice-averaged three-body problem (S2) was solved. The third-body motion has been assumed to be perturbed.

Consider the Cartesian coordinate system $X_1Y_1Z_1$ fixed to the orbital plane of a CBS (Fig. 2). The axes X_1 and Y_1 lie within the orbital plane, X_1 is directed to the periastron, and Z_1 coincides with the orbital normal n. Let φ be the angle measured from X_1 to the line of intersection of the orbital plane (viz., to the ascending node η of the third-body orbit), and let ϵ be the interinclination of the orbits (i.e., the angle between the normals n and n'). Then the direction cosines of n' are

$$Q = \sin\epsilon \sin\varphi , \tag{6a}$$

$$S = -\sin\epsilon \cos\varphi , \tag{6b}$$

$$N = \cos\epsilon \tag{6c}$$

with respect to the axes X_1, Y_1 and Z_1.

The perturbative functions for S2 were found in the form (only the first terms are presented)

$$R^{**} = \frac{\pi^2 q'\mathfrak{M}_1}{2(1-e'^2)^{3/2}} \frac{a^2}{a'^3} [\cdots] , \tag{7}$$

$$R_{tb}^{**} = \frac{\pi^2 q\mathfrak{M}_1}{2(1-e'^2)^{3/2}} \frac{1+q+q'}{(1+q)^2} \frac{a^2}{a'^3} [\cdots] , \tag{8}$$

where $[\cdots] = [3(1-e^2)N^2 - 15e^2Q^2 + 6e^2 - 1]$.

It should be noticed that the functions R^{**} and R_{tb}^{**} depend upon the third-body orbit orientation relative to the periastron of the CBS.

According to the Lagrange equations, we get

$$\frac{d\omega_{\rm ph}}{dt} = A\left[2 - 5Q^2 - N^2 + \cos i_{\rm ph}\left(\frac{1+4e^2}{1-e^2}\,Q\sin\omega_{\rm ph} + S\cos\omega_{\rm ph}\right)\right], \quad (9)$$

$$\frac{de}{dt} = 5AQS\,, \quad (10)$$

$$\frac{di_{\rm ph}}{dt} = AN\left(\frac{1+4e^2}{1-e^2}\,Q\cos\omega_{\rm ph} - S\sin\omega_{\rm ph}\right), \quad (11)$$

where

$$A = \frac{3\pi(1-e^2)^{1/2}P}{2(1-e'^2)^{3/2}P'^2}\,\frac{q'}{1+q+q'}\,.$$

Let us now turn to the period difference $\Delta P^{\rm tb}$ resulting from the third-body perturbation in the orbital elements of the CBS, and mainly in $\omega_{\rm ph}$ and e. Using formulae (6a)–(6c), (9), and (10), we get (Khodykin & Zakharov 1990)

$$\Delta P^{\rm tb} = \frac{3P^3e(1-e^2)^2}{2P'^2(1-e^2\sin^2\omega_{\rm ph})^2}\,\frac{\sin\omega_{\rm ph}}{(1-e'^2)^{3/2}}\,\frac{q'}{1+q+q'} \times\left\{2+\sin^2\epsilon\left[5\,\frac{\sin(2\varphi+\lambda)}{\sin\lambda}-3\right]\right\}, \quad (12a)$$

where $\cos\lambda = \cos\omega_{\rm ph}/(1-e^2)$, $0° < \lambda < 180°$. The apparent singularity in $\Delta P^{\rm tb}$ at $\omega_{\rm ph} = 180° \times j$ $(j = 0, 1, \ldots)$ seems to be easily removed:

$$\Delta P^{\rm tb} = (-1)^j\,\frac{15P^3e(1-e^2)}{2P'^2(1-e'^2)^{3/2}}\,\frac{q'}{1+q+q'}\,\sin^2\epsilon\,\sin 2\varphi\,. \quad (12b)$$

Although expression (12a) is rigorously correct only for an inclination $i_{\rm ph} = 90°$, it is quite accurate for the most of the far-separated eclipsing binaries. For DI Her, the values of φ and ϵ, provided that $\Delta P^{\rm tb}(\varphi, \epsilon) < 0$, are plotted in Figure 3.

The variations of orbital elements e, e', $i_{\rm ph}$, $i'_{\rm ph}$, etc., have been determined by the same numerical integration scheme. The classical and relativistic effects were accounted for in the periastron motion computation. Since for the binary motion the third-body perturbations, tidal-rotational, and GR effects are of the same order and small enough, they have been assumed to be additive and independent. The total period difference ΔP was computed as in the S1 case. The integration interval was chosen to exceed a few apsidal periods U (an overall return time of the apsidal line).

5. RESULTS

The numerical calculations of the three-body model have been carried out for the values of $\mathfrak{M}' = 0.01$, 0.1, 0.5, 0.78, 1.5, and 2.0 $\mathfrak{M}_\odot$. To test the self-consistency of S0 and S1, S1 and S2 solutions, the orbital elements' variations in CBSs have been compared at $t = lP$, $t = mP'$, respectively (l, m integers) for the several space orientations of the orbits. In the case of small mutual inclination ϵ no solutions were found which satisfy the observations. This result agrees with Guinan and Maloney's result, obtained from the analysis of Brown's relations. The desired effect ($\Delta P^{\rm tb}$) occurred at interinclination ϵ more than 21.°2. It is caused by the periastron regression or/and by a decrease in eccentricity.

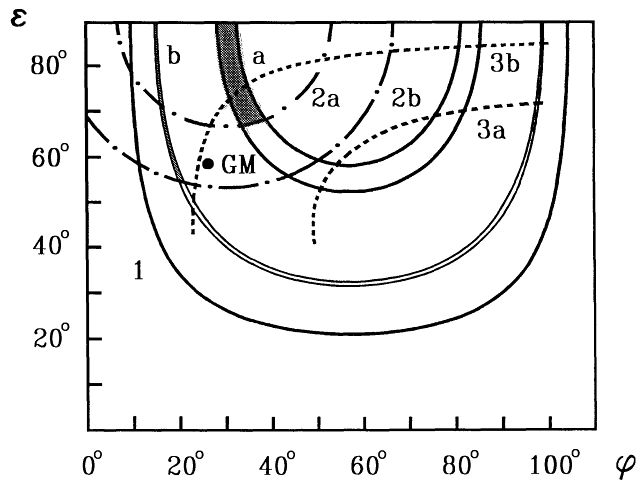

FIG. 3.—The (φ, ϵ)-plane: φ is the angle between the periastron and the line of intersection of the orbital planes $\eta\eta'$, and ϵ is the orbital interinclination. Curve 1 limits the area where the third body's effect $\Delta P^{\rm tb}$ is less than zero. As examples, the solution areas (*hatched*) for $\mathfrak{M}' = 1.0\ \mathfrak{M}_\odot$, $P' = 9.2$ yr (*a*) and $P' = 5$ yr (*b*) are given. The lines restricting the areas (*a*) and (*b*) are drawn according to the following conditions: the light-term effect is equal to 0.002 days (2*a* and 2*b*); the perturbation in the orbital inclination $\Delta i^{\rm cr}_{\rm ph}$ is equal to 0.°5 throughout $t_{\rm obs}$ (3*a* and 3*b*). The point labeled GM marks the Guinan and Maloney's solution.

Consider now the conditions restricting the areas of solutions (Fig. 4; and Fig. 5, $e' = 0$ and $\epsilon = 90°$, 60°):

1. The solutions for a massive third component ($\mathfrak{M}' \geq 0.78\ \mathfrak{M}_\odot$) are restricted by the Röemer delay (curves 5 in Figs. 5*a*, and 5*b*). This delay is equal to zero when the third-body orbit is close to the visual plane ($\epsilon \approx 90°$, $\varphi \approx 30°$ for DI Her). For such a case the following approximate expression for P' and $\mathfrak{M}'$ can be obtained from relation (12a):

$$P' \approx 30\left(\frac{\mathfrak{M}'}{\mathfrak{M}_1 + \mathfrak{M}_2 + \mathfrak{M}'}\right)^{1/2}\ ({\rm yr})\,. \quad (13)$$

2. The short-period solutions are restricted according to the condition $t_{\rm eff} \geq t_{\rm obs}$ (curves 4 in Figs. 5*a* and 5*b*). The duration of the third-body effect was found to depend weakly enough on the orbits' interinclination.

3. The variation of the binary's orbital inclination $i_{\rm ph}$ during $t_{\rm obs}$ is within the observational errors and is unable to distort the shape of the light curve. This requirement restricts the solutions' areas at some values of $\varphi = \varphi_{\rm crit}$ (Fig. 4), which are independent of mass $\mathfrak{M}'$ and can be found from formulae (11) and (12a):

$$\varphi_{\rm crit} = 92.°5,\ 79°,\ 62.°5,\ \text{and}\ 33°$$

at $\epsilon = 80°$, 70°, 60°, and 50°, respectively. Thus, this restriction diminishes the area of third-body parameters $\{\mathfrak{M}', P', \varphi\}$ at interinclinations ϵ less than 60°.

To summarize the restrictions mentioned above, we see that the area of solutions $\{\mathfrak{M}', P', \varphi\}$ (Figs. 4*a* and 4*b*) and $\{\mathfrak{M}', P'\}$ (Figs. 5*a* and 5*b*) vanishes rapidly with decreasing ϵ. At $\epsilon < 50°$ no appropriate solutions were found. It should be noted that the solutions restricted according to conditions 1–3 (see Figs. 3, 4, 5) correspond to the stable three-body systems. For a comparison the stability criteria of triple systems calculated according to Harrington (1977), Szebehely & Zare (1977), Hills (1983; coplanar case), and Roy (1979; three-dimensional case) are shown in Figure 5 (curves 1, 2, 3a and 3b).

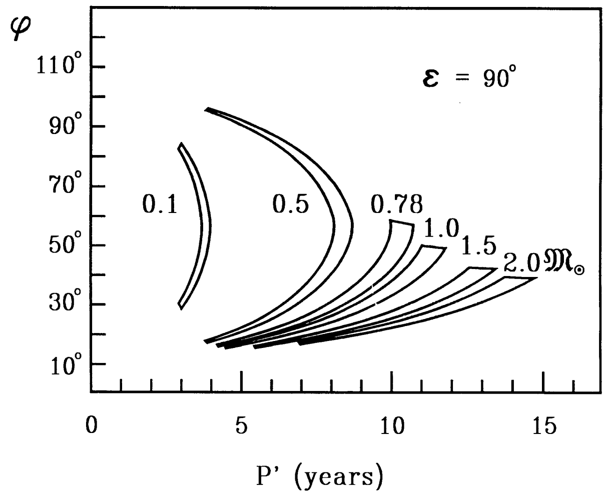

FIG. 4a

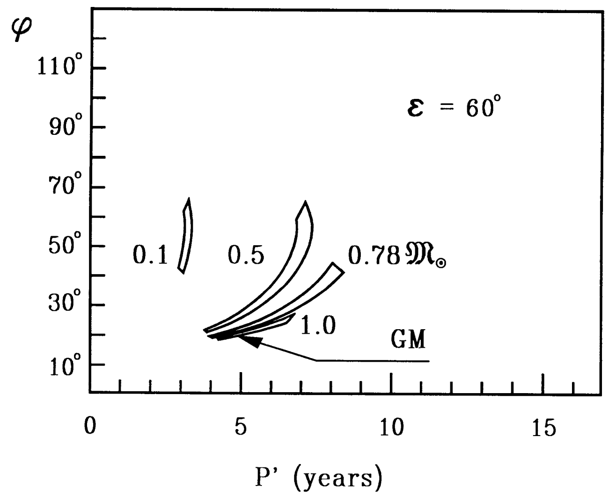

FIG. 4b

FIG. 4.—(*a*) Space of solutions—parameters $\{\mathfrak{M}', P', \varphi\}$, providing the desired effect $\Delta P^{\rm tb} = -1\overset{s}{.}68 \pm 0\overset{s}{.}15$ ($\epsilon = 90°$, $e' = 0$). The masses are given in solar masses ($\mathfrak{M}_\odot$). All the restrictions have been taken into account. The areas corresponding to the masses $\mathfrak{M}' \geq 0.78\ \mathfrak{M}_\odot$ are restricted by the Röemer delay (0.002 days). The short-period solutions are limited because of the following requirement: the third-body's effect's duration $t_{\rm eff}$ must exceed the interval of the binary's observation, $t_{\rm obs}$. (*b*) The space of solutions—parameters $\{\mathfrak{M}', P', \varphi\}$ as in (*a*), but for $\epsilon = 60°$. At large values of φ the areas are restricted according to the requirement that the perturbation in the orbital inclination not exceed the critical value of $\Delta i_{\rm ph}^{\rm cr} = 0\overset{\circ}{.}5$ during $t_{\rm obs}$ (i.e., the photometric light curve keeps the shape during $t_{\rm obs}$). The point labeled GM corresponds to the Guinan-Maloney solution ($\mathfrak{M}' = 1.0\ \mathfrak{M}_\odot$, $P' = 5$ yr, $\epsilon = 59°$).

An example of such a stable motion ($\mathfrak{M}' = 1.0\ \mathfrak{M}_\odot$, $P' = 10.5$ yr, $e' = 0.0$, $\epsilon = 80°$, $\varphi = 40°$) is given in Figure 6. The third-body orbit precesses slowly and nutates ($T_{\rm nut} = 0.5U \approx 4100$ yr). The binary eccentricity varies periodically from 0.422 to 0.587. At present ($t = 0$) the eccentricity decreases at $de/dt \approx -10^{-4}$ yr^{-1}, and the longitude of periastron increases at $d\omega_{\rm ph}/dt \approx 0.044$ yr^{-1}. These combined variations of e and $\omega_{\rm ph}$ provide the desired shift of the secondary minimum relative to the primary. The orbital inclination $i_{\rm ph}$ changes insignificantly: $di_{\rm ph}/dt \leq 2 \times 10^{-5}$ deg yr^{-1}. However, after $\approx 5 \times 10^4$ yr eclipses will no longer be visible from Earth.

6. DISCUSSION AND CONCLUSIONS

If the accuracy of the light-curve solution is too low to determine the periastron advance directly, the only way to evaluate the AMR in an eclipsing binary is to treat the variations of the secondary minimum phase ϕII (or the difference of the periods ΔP). With the presence of a third body, all the orbital elements

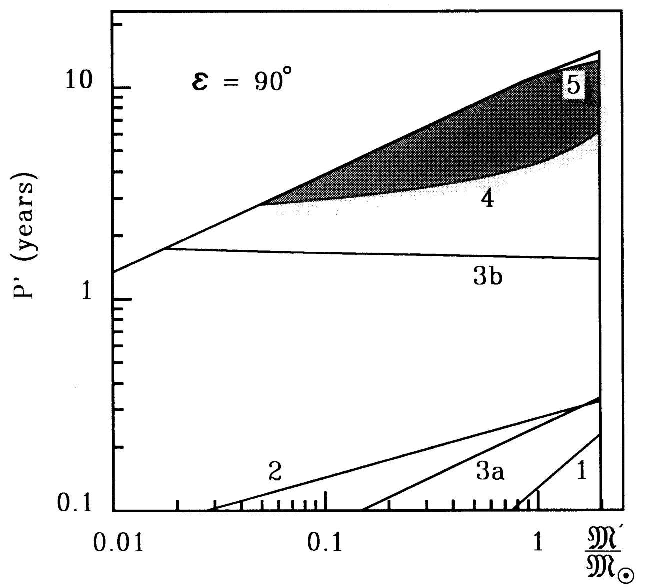

FIG. 5a

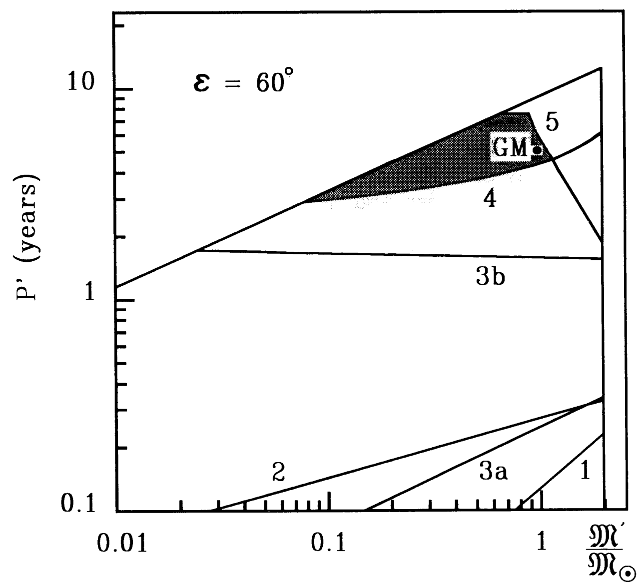

FIG. 5b

FIG. 5.—(*a*) $\{\mathfrak{M}'\text{-}P'\}$ solution area (*hatched*) for $\epsilon = 90°$ and $e' = 0$. Lines 1–3 indicate the boundaries of the stable motion according to stability criteria for triple systems: 1, Harrington (1977); 2, Szebehely & Zare (1977), Hills (1983) (coplanar case); 3, Roy (1979) (*a*: the binary's stability; *b*: the third-body motion stability, three-dimensional case). Curves 4 and 5 are plotted according to the following conditions: (4) $t_{\rm eff} = t_{\rm obs}$; (5) the amplitude of the light-term effect is 0.002 days. (*b*) $\{\mathfrak{M}'\text{-}P'\}$ solution area (*hatched*) at $\epsilon = 60°$, $e' = 0$. Curves 1–5 have the same meaning as in (*a*). The point labeled GM corresponds to the Guinan-Maloney solution: $\mathfrak{M}' = 1.0\ \mathfrak{M}_\odot$, $P' = 5$ yr, $\epsilon = 59°$.

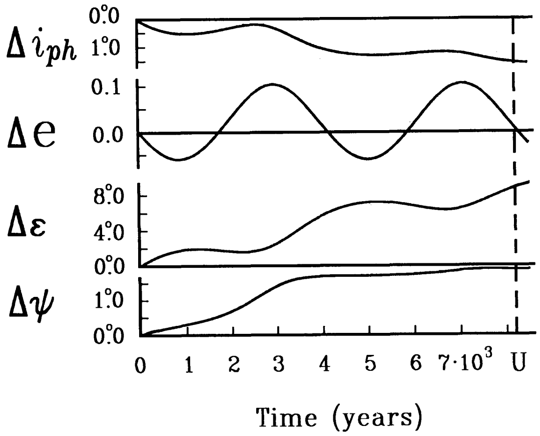

FIG. 6.—Example of stable hierarchical motion: $\mathfrak{M}' = 1.0\ \mathfrak{M}_\odot$, $P' = 10.5$ yr, $e' = 0$. The initial orbital interinclination ϵ is 80°. The binary orbit's inclination decreases slightly; the eccentricity e varies periodically. The nodal line of the orbits nutates with the period $T_{\rm nut} = 0.5U$ and precesses slowly. The apsidal period is $U \approx 8200$ yr.

vary, and, therefore, this method can lead to incorrect values of AMR$^{\rm obs}$ (and, consequently, to the wrong internal density concentration parameter k_2). In this paper, we suggested that the discrepancy between AMR$^{\rm obs}$ and AMR$^{\rm th}$ for DI Her can be explained by the third-body hypothesis.

The third-body effect $\Delta P^{\rm tb}$ was found to result from the variations of both the eccentricity and/or the periastron longitude. Up to now the case of a decrease in eccentricity has not been considered apposite because of the possible rapid circularization of the orbit. We found that the eccentricity variations depend on the third-body orbit's orientation relative to the apsidal line of the CBS. In a case of the third-body orbit's precession and of the apsidal motion in the CBS, e varies periodically. As a result the CBS retains its eccentric orbit. This conclusion seems to be important because it allows us to avoid the difficulties concerning the circularization in the CBS. Unfortunately, all the high-accuracy observational data obtained did not allow us to derive exactly the parameters ($\mathfrak{M}'$, P') and the orbital elements (e', ω', etc.) of the third companion. Therefore, only the region of possible solutions was found.

If the invisible member of the triple system is a relativistic object, then its mass may exceed the upper limit 1.7 $\mathfrak{M}_\odot$. According to equation (13), its orbital period P' is limited by 30 yr. In this case the third-body orbit must be close to the visual plane, which seems, however, to be unlikely.

A few words about the stability and evolution of the hierarchical triple system discussed in this paper. Of course these problems are too complex to be resolved here. The triple system stability criteria in the coplanar case were obtained by many authors, in particular by Hénon (1976), Harrington (1977), Szebehely & Zare (1977), and Hills (1983). Hadjidemetriou (1981) pointed out that there are no strict stability criteria in the three-dimensional case. We have used the empirical stability criteria of Roy (1979). It is clear from Figures 5*a* and 5*b* that the requirements of the third-body motion stability (curves 1, 2, 3a, and 3b) are less severe than those of the effect duration (curve 4) and of the light-curve limit distortion. The point labeled GM (Figs. 4*b* and 5*b*) corresponds to the stable Guinan-Maloney solution ($\mathfrak{M}' = 1.0\ \mathfrak{M}_\odot$, $P' = 5$ yr, $\epsilon = 59°$).

As for the triple system formation, we do not exclude the possibility of both capture and exchange processes. If the total energy of the binary and a single field star is negative, a bound triple system might be formed temporarily or be hierarchical and stable enough (Valtonen & Aarseth 1977). In any case, the components' ages would be different and the triple system stability constraints could be reduced or omitted. Valtonen (1976) and Hills (1983) showed that a massive binary can capture a low-mass field star and increase its semimajor axis and eccentricity (the final value of e is approximately 0.67). DI Her is known to be a well-separated system, and we have found that the orbital eccentricity might change periodically over a wide range of values. According to Hills's conclusions (1977), the majority of well-separated massive binaries with large eccentricities in the solar vicinity might be formed by an exchange process—the outcome of the stellar scattering. This result has been confirmed by Hut & Bahcall (1983) in a series of numerical orbit calculations: after the encounter of a binary and a massive field star, the lighter component of an initial binary is ejected onto an elongated eccentric orbit, with little interaction with the remaining massive binary. Certainly knowledge of the properties and the evolutionary state of DI Her components needs to be improved.

Let us consider now the possibility of the detection of the third body in DI Her. Taking into account that the partial luminosity of the third body L_v' is assumed to be less than 0.03, its V-magnitude will be over 12.2 mag (the V-magnitude of the eclipsing binary is 8.39 mag). Adopting the semimajor axis a' of the relative orbit to be less than 13 AU and the distance to DI Her to be $d \approx 500$ pc, one could evaluate the maximal angular distance between the third star and the binary $\rho \leq 0\overset{''}{.}02$. We think that such a faint companion could not be detected even by speckle interferometry.

The most reliable confirmations of the third-body hypothesis would be the detection of the periodic light-time effects in $O-C$ residuals (for both primary and secondary minima) and, on the other hand, the consistency of the theoretically predicted rates of periastron advance and of eccentricity decrease with those yielded by the direct observations. Therefore, further observations of this eclipsing system are urgently needed to improve the values of the spectroscopic orbital elements, and to obtain after a few years one more high-accuracy photoelectric light curve.

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