The Astrophysical Journal, 615:506–511, 2004 November 1

STABILITY OF TRIPLE STAR SYSTEMS WITH HIGHLY INCLINED ORBITS

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Received 2003 February 16; accepted 2004 July 8

ABSTRACT

It is well established that certain detached eclipsing binary stars exhibit apsidal motions whose values are in disagreement with calculated deviations from Keplerian motion based on tidal effects and the general theory of relativity. Although many theoretical scenarios have been demonstrated to bring calculations into line with observations, all have seemed unlikely for various reasons. In particular, it has been established that the hypothesis of a third star in an orbit almost perpendicular to the orbital plane of the close binary system can explain the anomalous motion at least in some cases. The stability of triple star systems with highly inclined orbits has been in doubt, however. We have found conditions that allow the long-term stability of such systems, so that the third-body hypothesis now seems a likely resolution of the apsidal motion problem. We apply our stability criteria to the cases of AS Cam and DI Her and recommend observations at the new Keck interferometer, which should be able to directly observe the third bodies in these systems.

Subject headings: binaries: eclipsing — celestial mechanics — instabilities — stars: individual (AS Camelopardalis, DI Herculis)

1. INTRODUCTION

Discrepancy between observation and theoretical predictions of the apsidal motion of certain detached binary stars has remained an outstanding problem for two decades. In the cases of AS Cam and DI Her, for example, observed apsidal motion rates are a fraction of the theoretical predictions based on stellar structure, tidal, and relativistic effects.

As was first pointed out by Rudkjøbing (1959), the effect of relativistic gravity is significant in the case of a number of detached binary stars. Although he considered only DI Her in detail, the number of interesting cases has grown to about half a dozen (Koch 1977; Moffat 1984), including AS Cam (Maloney et al. 1989).

The discovery of anomalous apsidal motion by Martynov & Khaliullin (1980) was initially considered a possible challenge to general relativity. Moffat (1984) invented an alternative gravity theory that harbored differing predictions for the apsidal motion of binary stars and yet maintained agreement with other tests of general relativity. The predictive power of this theory is weakened by the existence of a new adjustable parameter for each star. Moreover, increasingly severe tests of general relativity such as in Taylor & Weisberg (1989) make such large deviations at AU scales seem unlikely. Several other, less exotic, solutions have been proposed. In one scenario the rapid circularization of the orbit occurs because of dissipation of angular momentum from stellar oscillations or a large amount of mass loss. Another reasonable guess is that the close binary system (CBS) orbit is surrounded by a resisting medium in the form of gas clouds. The required density well exceeds observational limits in the case of DI Her, however. These and other alternatives are reviewed in Guinan & Maloney (1985), Maloney et al. (1989), and Claret (1997, 1998). It is the purpose of this paper to demonstrate the feasibility of one particular explanation that has been put forward in the literature. Our considerations may also find application in other triple star or suspected triple star systems such as in Coe et al. (2002), where a triple star model for the X-ray pulsar AX J0051−733 is proposed to explain puzzling features of the spectrum of a candidate optical counterpart.

It is well known that the hypothesis of third stars in outer orbits of these close binary systems can bring theory into line with observed apsidal periods (Khaliullin et al. 1991; Khodykin & Vedeneyev 1997), but the stability of such triple star systems has been in doubt (Harrington 1968). We show here that the inclusion of the apsidal motion as an additional perturbation leads to the conclusion that such triple star motions can be stable. In particular, the triple star models of Khodykin & Vedeneyev (1997) and Khaliullin et al. (1991), which reconcile the cases of AS Cam and DI Her with observations, are shown to be stable. Furthermore, we show that observations utilizing the new Keck interferometer should be able to directly image the putative third bodies in these two systems.

2. THE LAGRANGE PLANETARY EQUATIONS FOR THE HIERARCHICAL THREE-BODY SYSTEM

We have studied numerically the dynamical evolution of a hierarchical triple system consisting of a massive CBS and a third star of moderate mass. Figure 1 shows the notation used. The calculations were done perturbatively using the disturbing function method (Kopal 1978). We have assumed that the three stars are pointlike and isolated from other stars. We ignore internal dynamical exchanges such as synchronization, angular

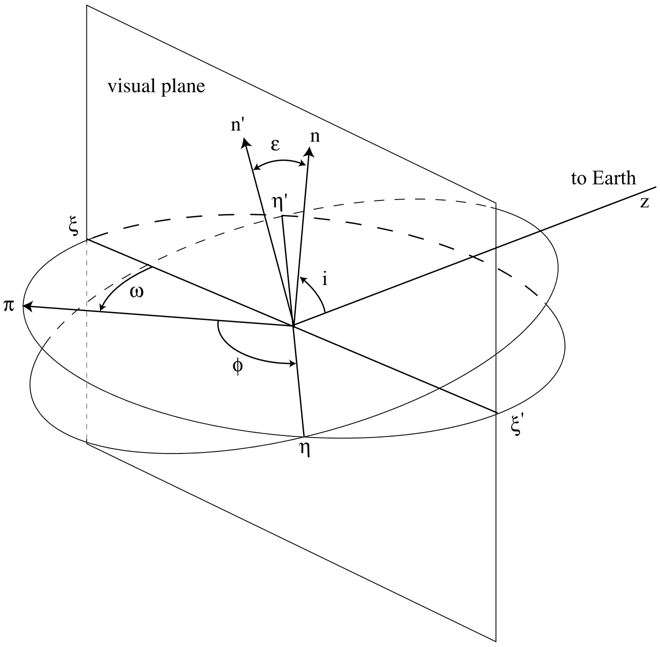

FIG. 1.—Kinematic variables describing the relative orientation of the orbits of the CBS and the third body.

momentum exchange, and orbital precession of the CBS. Classical tidal effects and relativistic effects are assumed to be independent and additive.

The disturbing functions for the CBS and the third body are adapted from Brown & Shook (1933, p. 14). They are, respectively,

$$R = 4\pi^2 \frac{m_3}{r}\left(\frac{r}{r'}\right)^3 \sum_{n=1}^{\infty} \frac{m_1^n - (-m_2)^n}{(m_1+m_2)^n}\left(\frac{r}{r'}\right)^{n-1} P_{n+1}, \quad (1)$$

$$R^{\rm tb} = \frac{m_1+m_2+m_3}{(m_1+m_2)^2}\frac{m_1 m_2}{m_3} R. \quad (2)$$

We average R and $R_{\rm tb}$ over the mean anomalies of the CBS and third body (hence, twice averaged). The first-order terms of the twice-averaged disturbing functions, R_2 and $R_2^{\rm tb}$, are

$$R_2 = \frac{\pi^2 q' m_1 a^2}{2(1-e'^2)^{3/2} a'^3}\left[3N^2\left(1-e^2\right) - 15e^2Q^2 + \left(6e^2-1\right)\right], \quad (3)$$

$$R_2^{\rm tb} = \frac{q(1+q+q')}{q'(1+q)^2} R_2, \quad (4)$$

where the masses are included via the ratios $q = m_2/m_1$ and $q' = m'/m_1$. We choose our units of measurement to be AU, years, and solar masses, so that the Newtonian gravitational constant is $G = 4\pi^2$. The orientation of the third-body orbital plane with respect to the close binary orbital plane is described by the direction cosines (Q, S, N) of the unit vector normal to the third-body orbital plane. We refer the direction cosines to the periastron, a perpendicular to the periastron, and the direction normal to the close binary orbit. Let ϵ be the angle between the two orbital planes, and call the angle measured from the periastron to the line of intersection of the orbital planes ϕ. We then have

$$Q = \sin\epsilon \sin\phi, \quad (5)$$

$$S = -\sin\epsilon\cos\phi, \quad (6)$$

$$N = \cos\epsilon. \quad (7)$$

Since we are averaging over the mean anomaly, there is no Lagrange planetary equation for M, and furthermore the semimajor axis a has no time dependence. The planetary equations for the perturbation of the remaining CBS orbital elements by the third body are

$$\left(\frac{da}{dt}\right)_{\rm tb} = 0, \quad (8)$$

$$\left(\frac{de}{dt}\right)_{\rm tb} = 5AeQS = -\frac{5}{2}Ae\sin^2\epsilon\sin 2\phi, \quad (9)$$

$$\left(\frac{d\omega}{dt}\right)_{\rm tb} = A\left[2 - 5Q^2 - N^2 - N\cot i\left(\frac{1+4e^2}{1-e^2}Q\sin\omega + S\cos\omega\right)\right], \quad (10)$$

$$\left(\frac{di}{dt}\right)_{\rm tb} = AN\left(\frac{1+4e^2}{1-e^2}Q\cos\omega - S\sin\omega\right), \quad (11)$$

$$\left(\frac{d\Omega}{dt}\right)_{\rm tb} = AN\csc i\left(S\cos\omega + \frac{1+4e^2}{1-e^2}Q\sin\omega\right), \quad (12)$$

where

$$A = \frac{3\pi(1-e^2)^{1/2}Pq'}{2(1-e'^2)^{3/2}P'^2(1+q+q')}. \quad (13)$$

The planetary equations for the perturbation of the third-body orbital elements are

$$\left(\frac{da'}{dt}\right)_{\rm tb} = 0, \quad (14)$$

$$\left(\frac{de'}{dt}\right)_{\rm tb} = 0, \quad (15)$$

$$\left(\frac{d\omega'}{dt}\right)_{\rm tb} = B\left[\frac{3e^2+2}{2(1-e^2)} - \frac{3}{2}\left(\frac{1+4e^2}{1-e^2}Q^2 + S^2\right) + \cot i'\left(\frac{1+4e^2}{1-e^2}QT + SU\right)\right], \quad (16)$$

$$\left(\frac{di'}{dt}\right)_{\rm tb} = -B\left(\frac{1+4e^2}{1-e^2}QF + SG\right), \quad (17)$$

$$\left(\frac{d\Omega'}{dt}\right)_{\rm tb} = -B\csc i'\left(\frac{1+4e^2}{1-e^2}QT + SU\right), \quad (18)$$

where

$$B = \frac{3\pi q(1-e^2)P^{4/3}}{2(1-e'^2)^2 P'^{7/3}(1+q)^{4/3}(1+q+q')^{2/3}}. \quad (19)$$

In writing equations (16)–(18) we have introduced the direction cosines of the nodal line of the third body referred to the same CBS axes by which Q, S, and N are defined above. These cosines are

$$F = \sin(\Omega' - \Omega)\cos i\sin\omega + \cos(\Omega' - \Omega)\cos\omega, \quad (20)$$

$$G = \sin(\Omega' - \Omega)\cos i\cos\omega - \cos(\Omega' - \Omega)\sin\omega, \quad (21)$$

$$H = \sin(\Omega' - \Omega)\sin i. \quad (22)$$

Finally, we use cosines of the direction perpendicular to the third-body nodal line and behind the visual plane. These cosines are

$$T = [\cos(\Omega' - \Omega)\cos i \cos i' + \sin i \sin i'] \sin\omega - \sin(\Omega' - \Omega)\cos i' \cos\omega, \quad (23)$$

$$U = [\cos(\Omega' - \Omega)\cos i \cos i' + \sin i \sin i'] \cos\omega + \sin(\Omega' - \Omega)\cos i' \sin\omega, \quad (24)$$

$$V = \cos(\Omega' - \Omega)\sin i \cos i' - \cos i \sin i'. \quad (25)$$

3. AN ALTERNATIVE FORMULATION OF THE EQUATIONS OF MOTION

It is worth noting that the equations of motion can be written in a particularly elegant fashion by noting that they determine nothing more than an instantaneous rotation that can be referred to the CBS coordinates. The problem is then to express $d\omega/dt$, di/dt, $d\phi/dt$, and $d\epsilon/dt$ in terms of this angular velocity vector (Goldstein et al. 2002, p. 174). Referring to this angular velocity as $\boldsymbol{\Psi}$, we first resolve it into three components: $\boldsymbol{\Psi} = \boldsymbol{\Psi}_\omega + \boldsymbol{\Psi}_i + \boldsymbol{\Psi}_z$. These components are related by rotation(s) to the angular velocity components. For example, $\boldsymbol{\Psi}_i$ is just $-di/dt$ referred to the nodal line $\xi\xi'$. Referring to Figure 1, we see that a rotation by ω about the z-axis brings this angular velocity into the CBS system; thus

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & & 1 \end{pmatrix} \begin{pmatrix} -di/dt \\ 0 \\ 0 \end{pmatrix}, \quad (26)$$

where X, Y, and Z are the components of $\boldsymbol{\Psi}$ in the CBS coordinates. Proceeding in this manner, one obtains three equations giving X, Y, and Z in terms of $d\omega/dt$, di/dt, and $d\Omega/dt$. These equations are inverted to yield

$$d\omega/dt = \cot i(X\sin\omega + Y\cos\omega) + Z, \quad (27)$$

$$di/dt = -X\cos\omega + Y\sin\omega, \quad (28)$$

$$d\Omega/dt = -\csc i(X\sin\omega + Y\cos\omega). \quad (29)$$

Similar considerations for a circular third-body orbit result in

$$d\epsilon/dt = X\cos\phi + Y\sin\phi, \quad (30)$$

$$d\phi/dt = -\cot\epsilon(X\sin\phi - Y\cos\phi) + Z. \quad (31)$$

Comparing equations (27)–(29) with equations (10)–(12), we determine the rate of rotation of the CBS frame, as measured by CBS coordinates, to be

$$(X, Y, Z) = -A\left(NQ\frac{1+4e^2}{1-e^2}, NS, N^2 + 5Q^2 - 2\right). \quad (32)$$

It has been pointed out by Kiseleva et al. (1998) (using different angles to define orbit orientations) that this formulation admits two exact integrals and leads to a first-order elliptical differential equation for the eccentricity. Our calculations, however, are entirely numerical and are based on equations (8)–(12) and (14)–(18).

4. THE INSTABILITY PROBLEM OF HIERARCHICAL TRIPLE STAR SYSTEMS

There are a couple of ways of seeing the problem of instability. One approach utilizes the conservation of angular momentum. We assume that $i \approx \pi/2$, so that the $\cot i$ term in equation (10) can be ignored. This leads to a large eccentricity excursion due to the angular momentum exchange between the CBS and the third body. Since the disturbing function $R_2^{\rm tb}$ depends on neither M' (the mean anomaly) nor ω' (the longitude of the periastron of the third body with respect to the ascending node), there is no secular variation of the third-body semimajor axis a' and eccentricity e'. Therefore, the orbit of the third body maintains its shape, and the magnitude of its orbital angular momentum $\boldsymbol{L}'$ is a constant of motion. The direction of the third-body orbital angular momentum, however, will change, as the following argument shows. The orbital angular momentum of the close binary system $\boldsymbol{L}_{\rm BS}$ is 3–10 times smaller than $\boldsymbol{L}'$ for the systems under investigation. Since the rotational angular momentum of the stars is about 2 orders of magnitude less than $\boldsymbol{L}'$, the total angular momentum of the system is $\boldsymbol{L}_{\rm tot} = \boldsymbol{L}_{\rm BS} + \boldsymbol{L}'$. Calculations reveal that the CBS angular momentum is transferred to the third star. Since the magnitude of $\boldsymbol{L}'$ can not change, this angular momentum transfer forces a change in the orientation of the third-body orbit with respect to the total angular momentum. Conservation of angular momentum dictates the connection between ϵ and e shown in Figure 2. As angular momentum is transferred, the coordinate values (e, ϵ) slide along one of the integral curves determined from the initial values of the eccentricities e and e', mass ratios q and q', relative inclination ϵ, and ratio of the semimajor axes a/a'. In the presence of a small but nonzero $\cot i$ the above argument does not take the orbit all the way to $e = 0$, but the eccentricity may become close enough to unity that tidal effects or even collision destroy the system. This turns out to be the case for DI Her and AS Cam.

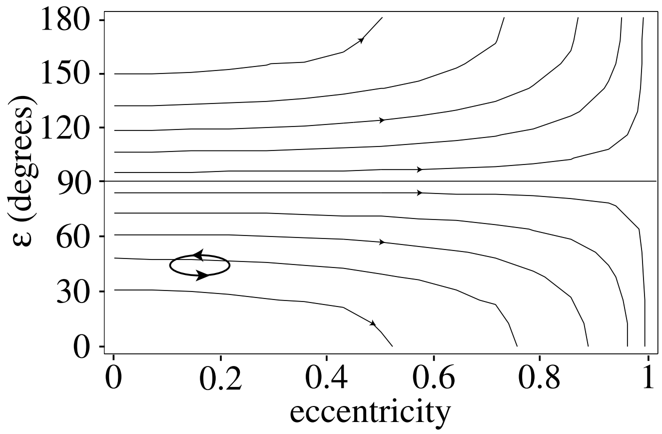

FIG. 2.—Evolution of ϵ and e due to angular momentum exchange between the CBS and the third body. The loop schematically illustrates the results of numerical integration of the equations of motion of AS Cam including the influence of the CBS apsidal motion as an additional perturbation.

We can gain additional insight into the dynamics of the eccentricity by looking directly at the equations of motion. More specifically, we look at equation (10) neglecting the $\cot i$ term and further assume that the motion of the nodal line $\eta\eta'$ caused by nutation and precession of the orbits is small. Under these assumptions,

$$\left(\frac{d\omega}{dt}\right)_{\rm tb} \approx A(2 - 5Q^2 - N^2), \quad (33)$$

$$\left(\frac{d\omega}{dt}\right)_{\rm tb} \approx -\left(\frac{d\phi}{dt}\right)_{\rm tb}. \quad (34)$$

First, we note that for $\epsilon \leq 30^\circ$ we have $Q, S \ll 1$. We see then that the right-hand side of equation (33) is always positive. Thus ω and ϕ change monotonically and run over all quadrants, never coming to rest. Therefore, the perturbations in the eccentricity (eq. [9]) are periodic, and the eccentricity suffers no excursion to a value nearing unity. The situation is much different if $\epsilon > 30^\circ$. At such inclinations, equation (33) implies in general four values of ϕ for which $d\omega/dt = 0$. Let us examine the motion of ϕ at these roots. Equations (33) and (34) together imply

$$\left(\frac{d^2\phi}{dt^2}\right)_{\rm tb} \sim \sin^2\epsilon \sin 2\phi \left(\frac{d\phi}{dt}\right)_{\rm tb}. \quad (35)$$

This tells us that the four points of stationary ω (and ϕ) will be stable only if $\sin 2\phi < 0$. Looking at equation (9), we see that this same condition guarantees $(de/dt)_{\rm tb} > 0$. Thus ϕ will wander until it reaches a value that results in an eccentricity excursion. Again, we emphasize that these semiquantitative arguments are limited to the extent that we neglect the $\cot i$ term in equation (10).

The characteristic timescale for the change of eccentricity can be obtained from equation (9):

$$\tau_e \approx \frac{1-e}{(de/dt)} \approx 0.1\sqrt{\frac{1-e}{1+e}}\frac{(1-e'^2)^{3/2}a'^3}{ePm'}. \quad (36)$$

For DI Her and AS Cam these times are about 700 and 400 yr, respectively, as was confirmed directly by numerical integration. Thus at first glance the conclusion seems to be that only nearly coplanar hierarchical triple systems can be stable for more than a few hundred years. If this conclusion were correct, the third-body hypothesis would be eliminated as a probable solution to the apsidal motion discrepancy.

5. A POSSIBLE RESOLUTION OF THE PROBLEM OF INSTABILITY

We now explore the consequences of including the effect of stellar structure (namely tidal-rotational deformation of the CBS pair) and the relativistic effect as additive perturbations on the motion of ω. We assume that the structure effect $(d\omega/dt)_{\rm cl}$ and the relativistic effect $(d\omega/dt)_{\rm rel}$ act in simple superposition with the effect of the third body $(d\omega/dt)_{\rm tb}$, so that their influence can be represented by simply adding them to $(d\omega/dt)_{\rm tb}$. If the combined effect of these two additional terms is of the same order or greater than the third-body effect, that is, if

$$\left(\frac{d\omega}{dt}\right)_{\rm tb} \lesssim \left(\frac{d\omega}{dt}\right)_{\rm cl} + \left(\frac{d\omega}{dt}\right)_{\rm rel}, \quad (37)$$

then the motion of ω will not stop. Thus ω and ϕ will change monotonically, resulting in periodic perturbations of the orbital elements of the CBS leading to stability, as in the case of low inclinations discussed above.

We can derive stability criteria on the basis of equation (37). We consider the cases in which either the classical deformation effect or the relativistic effect dominates using well-known relationships for $(d\omega/dt)_{\rm cl}$ from Kopal (1978) and $(d\omega/dt)_{\rm rel}$ from Rudkjøbing (1959) and Martynov & Khaliullin (1980).

TABLE 1
Stability Criteria for AS Cam and DI Her

System	$S^*_{\rm tb}$	$S_{\rm cl}$	$S^{**}_{\rm tb}$	$S_{\rm rel}$
AS Cam............	3×10^{-6}	4.4×10^{-6}	3.8×10^{-8}	10^{-8}
DI Her..............	4×10^{-7}	10^{-7}	$(0.7–1.5)\times10^{-8}$	10^{-8}

If the classical effect dominates in the sense that $|(d\omega/dt)_{\rm tb}| < (d\omega/dt)_{\rm cl}$, we have

$$S^*_{\rm tb} = \frac{(1-e^2)^7}{(1-e'^2)^{3/2}}\left(\frac{a}{a'}\right)^3\frac{q'}{q(1+q)}\left(\frac{3}{4}-\cos^2\epsilon\right) < S_{\rm cl} = 10k_2r^5. \quad (38)$$

If the relativistic effect is predominant $[|(d\omega/dt)_{\rm tb}| < (d\omega/dt)_{\rm rel}]$, then

$$S^{**}_{\rm tb} = \frac{(1-e^2)^{3/2}}{(1-e'^2)^{3/2}}\left(\frac{a}{a'}\right)^3\frac{q'a}{M_1(1+q)^2}\left(\frac{3}{4}-\cos^2\epsilon\right) < S_{\rm rel} = \frac{G}{c^2} = 10^{-8}. \quad (39)$$

Table 1 displays the application of these criteria to AS Cam and DI Her. We see that, according to our hypothesis, both of these supposed triple star systems are predicted to be stable. The stability of AS Cam is provided by the classical effect, whereas DI Her is stable because of the relativistic apsidal motion.

We have confirmed this behavior by means of numerical integrations of the equations of motion. The calculations were done perturbatively using the disturbing function method (Kopal 1978). We have assumed that the stars are pointlike, and the close encounters are ignored. We ignore internal dynamical exchanges such as synchronization, angular momentum exchange, and orbital precession of the CBS. The apsidal motion in the CBS caused by both the classical tidal-rotational deformation of the components and the relativistic apsidal motion is described by disturbing functions as in Khaliullin et al. (1991). The classical tidal effects and relativistic effects are assumed to be independent and additive. The results for AS Cam are shown in Figure 2. Thus the angular momentum is transferred back and forth between the third body and the CBS. These periodical variations in e and ϵ can be visualized as a flapping of the CBS and third-body orbits, almost like a butterfly (Fig. 3).

6. DISCUSSION

We begin our discussion by contrasting our stability criteria with those of Roy (1979), Szebehely & Zare (1977), and Eggleton & Kiseleva (1995) for the case of DI Her using the same orbital parameters as Khaliullin et al. (1991).

Roy (1979) allows a very close orbit of the third body, the restriction being only that the semimajor axis satisfy $a' \geq 0.3$ AU corresponding to a period $P' \geq 18.3$ days. This seems much too close to the inner binary for stability at any inclination of the third-body orbit.

Although Szebehely & Zare (1977) deal primarily with the case of coplanar orbits, they indicate how their results may be extended to third-body orbits inclined to the inner binary orbit. We have applied their stability criteria assuming that the third-body orbit is perpendicular to the inner binary orbital plane. The result is that stability criterion requires $a' \geq 2.9$ AU ($P' \geq 1.5$ yr). This seems more reasonable, but in fact numerical

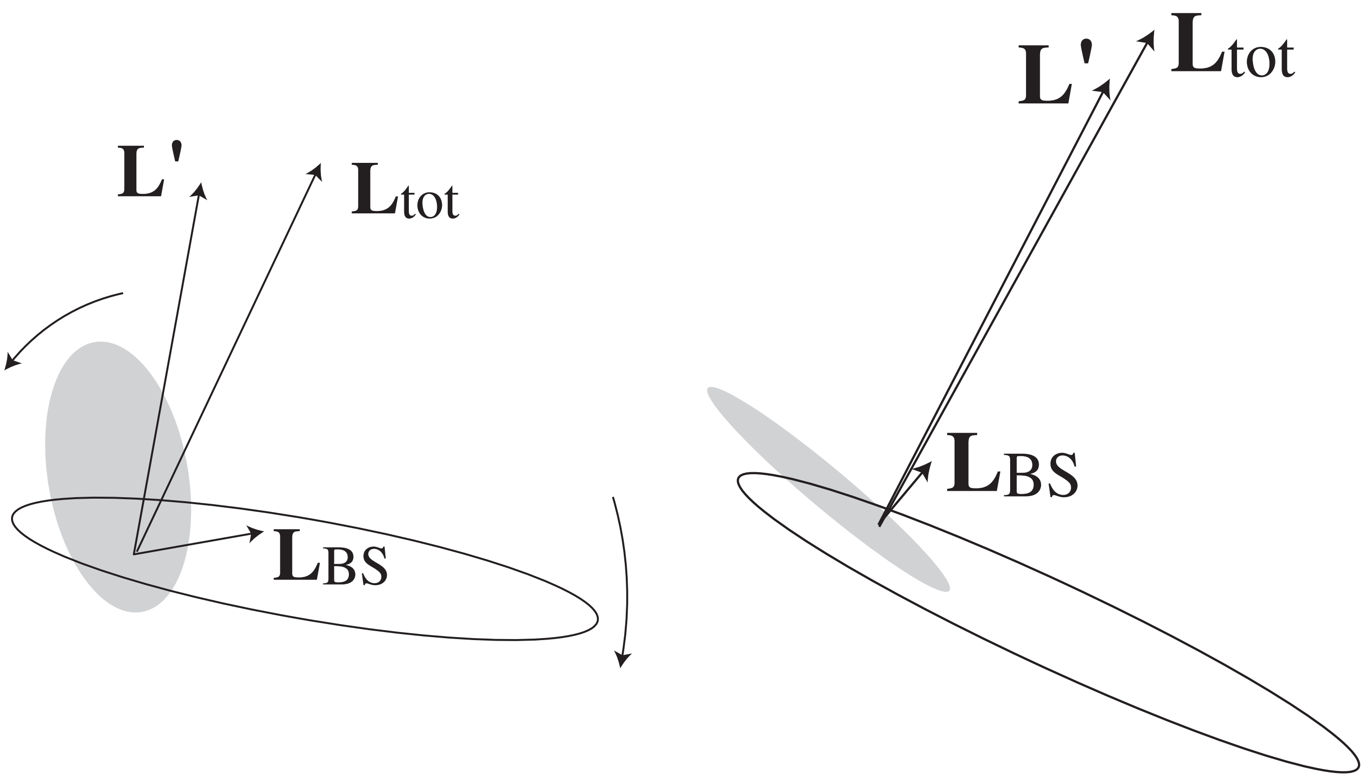

FIG. 3.—Motion schematic. As the CBS loses angular momentum, its orbit overturns and eccentricity increases to a value close to unity. If the apsidal motion of the CBS is included as an additional perturbation, then ϵ and e oscillations are moderated, and the triple system orbits move back and forth like a butterfly.

simulations suggest that this is still too close (Khodykin & Vedeneyev 1997).

Finally, Eggleton & Kiseleva (1995) predict that the system is stable for any third-body orbit with $a' \geq 1.2$ AU ($P' \geq 0.39$ yr).

The criteria we propose prove more restrictive. From equation (39) we determine that $a' \geq 9.5$ AU ($P' \geq 9$ yr). We have confirmed the stability of the purported three-body system of DI Her under this restriction on the third-body orbit. We also wish to emphasize that the criteria of Roy (1979), Szebehely & Zare (1977), and Eggleton & Kiseleva (1995) are all based on the purely Newtonian gravitational theory of point-mass orbits. Our criteria depend on structure effects and/or general relativity. In the case of DI Her the effect of general relativity dominates. It appears that the stability of the hypothetical three-body system of DI Her is to be found in the physics of general relativity rather than structure effects. Thus general relativity itself provides stability to the three-body model of DI Her, which seems necessary to bring its theoretical apsidal motion into line with observations. A similar stabilizing role for general relativity was found by Holman et al. (1997) in their investigations of a planet orbiting one star of a binary system. Our criteria are somewhat more general in that structure effects are also considered.

We close with a discussion of the prospects of making direct observations of these hypothetical third-body companions of AS Cam and DI Her in light of recent advances in optical interferometry and adaptive optics.

Indirect evidence for a third body in AS Cam (B+B9.5, $P = 3.43$ days, $e = 0.17$, $V = 8.6$) has already been found by Kozyreva et al. (1999) and Kozyreva & Khaliullin (1999), who found, imposed on the timing of eclipse minima, a cyclic variation with a period of 2.2 yr. They have interpreted this signal as due to the Roemer-like influence of a third star. The calculations of Khodykin & Vedeneyev (1997) indicate that in order to account for the anomalous apsidal motion of AS Cam, the third body should be of about 1 $M_\odot$. Using this estimate, binary masses of 3.3 and 2.5 $M_\odot$ (Hilditch 1972), and the 2.2 yr period, the semimajor axis of the orbit would be 3.2 AU. This corresponds to a light-travel time of 27 minutes. Combining this with the amplitude of 4.18 minutes measured by Kozyreva et al. (1999) yields an orbital inclination of 9° with respect to the line of sight, so that nearly the full 3.2 AU is visible to the observer. Assuming a distance of 480 pc, the maximum angular elongation is 0.″007. This is twice the resolution limit of the Keck interferometer operating at 1.5 μm with its 85 m baseline. The interpretation of eclipse timings by Kozyreva et al. (1999) and Kozyreva & Khaliullin (1999) also predicts the times of maximum elongation.

In the case of DI Herculis, no indirect indication of third light exists. However, information from past theoretical analyses provides hope that present interferometers should be capable of directly observing the putative third body. The analysis of Khaliullin et al. (1991) suggests that the minimum third-body mass is about 0.8 $M_\odot$ and that the minimum period is about 7 yr. Assuming binary masses of 5.15 and 4.52 $M_\odot$ (Popper 1982), we conclude that the semimajor axis is at least 8 AU. Since the orbit is expected to be highly inclined, and the distance to DI Her is about 500 pc, we expect a maximum angular elongation of 0.″02.

Of course, the ability to resolve these third bodies is of little use unless they are sufficiently bright. Here infrared observations are a great advantage, since the compact binary stars are relatively massive in comparison with the hypothesized third bodies. For example, applying the mass-luminosity relationship to AS Cam, we conclude that the third star should have a total luminosity less than the system by 4.3 mag. A simple calculation based on the Planck distribution and assumed temperatures of 20,000 and 6000 K for the binary and third star, respectively, predicts that, in the H (1645 ± 155 nm) and

K (2200 ± 480 nm) bands, the third star is dimmer by only about 1 mag. A similar calculation for DI Her predicts that in the H and K bands the third star is dimmer than the system by 3.5 mag. Finally, the magnitudes of these systems are such that the compact binary stars may serve as natural guide stars for adaptive optics in the case of the Keck interferometer.

Bolstered by indirect evidence in the case of AS Cam and dynamical stability indicated by the considerations of this paper, the case for a third-body solution to the long standing problem of anomalous apsidal motion is stronger than ever. The final judgment, however, may soon be expected from the current generation of interferometers.

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